

Game Theory

Thinking Strategically II: Nash Equilibria (Part 2)

Review:

Prisoners' Dilemma
Dominance: Strict vs Weak
Pareto-dominance
Common Knowledge
Best Response
Nash Equilibria

Review:

Common Knowledge
Best Response
Nash Equilibria

Common Knowledge

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Some fact (call it " p ") is *common knowledge* just in case
(1) everyone knows that p , (2) everyone knows that
everyone knows that p , (3) everyone knows that
everyone knows that everyone knows that p , ...

Best Response

Best Response

Given what all other players are doing, a strategy is a
best response just in case a player cannot do better by
switching to a different strategy.

Best Response

If you knew what the other player would do, you should play
your best response to their move.

Likewise, if the other player knew what you would do, they
should play their best response to your move.

You are trying to predict what *they* will do.

They are trying to predict what *you* will do.

Best Response

Sometimes (like in “Rock, Paper, Scissors”), there is no *stable* stopping point.

Example:



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Example:

		2	
		Poison A	Posion B
1	Drink A	Die, Live	Live, Die
	Drink B	Live, Die	Die, Live

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And develop an immunity to iocane powder

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Nash Equilibria

Nash Equilibria

Set of strategies, one for each player, such that no player has an incentive to change their strategy.

An Example

Nash Equilibria

(What's your best response?)

Example: What should you do if you think Player 2 will play l?

		2	
		l	r
1	U	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria

(What's your best response?)

Example: What should you do if you think Player 2 will play l?

		2	
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Nash Equilibria (What's their best response?)

Example: What should Player 2 do if you play D?

		2	
		l	r
1	U	2, 1	1, 2
	D	4, 5	0, 10

Nash Equilibria (What's their best response?)

Example: What should Player 2 do if you play D?

		2	
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1	U	2, 1	1, 2
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Nash Equilibria (What's your best response?)

Example: What should you do if Player 2 plays r?

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Nash Equilibria (What's your best response?)

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Nash Equilibria (What's their best response?)

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Nash Equilibria

Example: If you play U, Player 2 should play r.
And, if Player 2 plays r, you should play U.

		2	
		l	r
1	U	2, 1	1, 2
	D	4, 5	0, 10

U is the *best response* to r,
which is the *best response*
to U.

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Set of strategies, one for each player, such that each strategy is a *best response* to the others.

Nash Equilibria

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And, if Player 2 plays r, you should play U.

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Nash Equilibria are *stable* in the following sense:

If both players were to know that they were in one, they'd stay there.

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[Compare to Rock, Paper, Scissors]

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Nash Equilibria

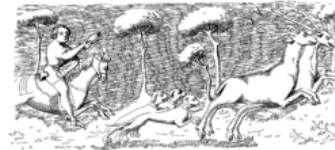
WHY?

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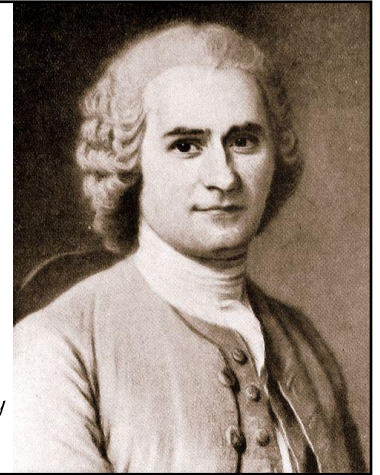
Example: Stag Hunt

Nash Equilibria

Example: Stag Hunt



Jean Jacques Rousseau (1712-1778)
The Discourse on the Origin of Inequality



Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3,3	0,2
	Hare	2,0	1,1

Nash Equilibria

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Activity: Investment Game

Nash Equilibria

Example: Investment Game

Players: you

Strategies: invest \$0 or invest \$10

Payoffs: if you invest \$0, you win/lose nothing
if you invest \$10, win \$11 if >90% invests
win \$0 otherwise.

Nash Equilibria

Example: Investment Game

		2	
		invest	refrain
1	Invest	1, 1	-10, 0
	Refrain	0, -10	0, 0

Nash Equilibria

Example: Bank Run



**Example:
Meeting Game**

Meeting Game (Stag Hunt)

		2	
		go	stay
1	Go	3, 3	0, 1
	Stay	1, 0	1, 1

Meeting Game (Stag Hunt)

Communication helps!

		2	
		go	stay
1	Go	3, 3	0, 1
	Stay	1, 0	1, 1

Example:
Stoplight

Nash Equilibria

Example: Stoplight game

		2	
		go	stop
1	Go	-5, -5	1, 0
	Stop	0, 1	-1, -1

Nash Equilibria

Example: Stoplight game

		2	
		go	stop
1	Go	-5, -5	1, 0
	Stop	0, 1	-1, -1

Example: The Prisoners' Dilemma

Nash Equilibria

		Them	
		A	B
You	A	0,0	2,-1
	B	-1,2	1,1

Nash Equilibria

		Them	
		A	B
You	A	0,0	2,-1
	B	-1,2	1,1